

**1. Logistic function**

Problem of KNN: 1. Need to store all data.

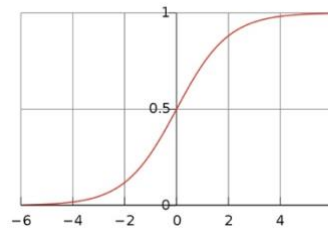
2. Need to calculate the distance matrix.

3. Slow predicting.

If we have a formula:  $w_h H + w_w W + w_0 \geq 0.5$

$$\frac{1}{1+e^{-(w_h H + w_w W + w_0)}} \geq 0.5$$

(If  $w_h$ ,  $w_w$ , and  $w_0$  are large)



$$\frac{1}{1+e^{-t}} \geq 0.5$$

Training: fit the training data (how to choose  $w_h$ ,  $w_w$  and  $w_0$ )

**Make  $\frac{1}{1+e^{-(w_h H + w_w W + w_0)}} \geq 0.5$  correct for the training data**

$$Y^{output} = \frac{1}{1+e^{-(w_h H + w_w W + w_0)}}$$

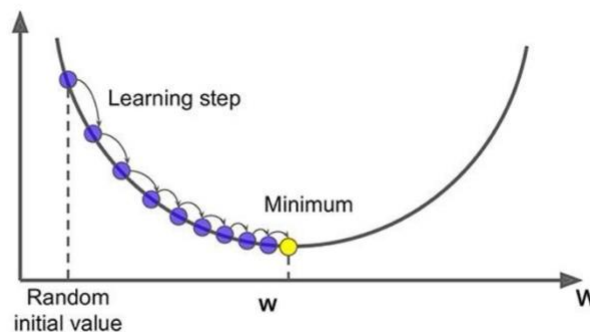
- ✧  $(Y^{output} - Y)^2$  should be as small as possible -> use calculus
- ✧ Y: the true label we have for training data
- ✧ It is the loss function we would like to minimise

**Gradient descent algorithm**

$$L = \sum_{P_1}^{P_4} (Y^{output} - Y)^2 \text{ is a function of } w$$

For each  $w$ , we want to find a value to make the function value **smallest**

$$\sum_{P_1}^{P_4} (Y^{output} - Y)^2$$



Step1. Initialise  $w_h$  and  $w_w$  and  $w_0$  at any random value

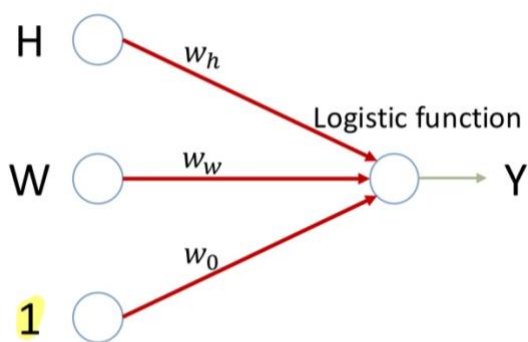
Step2. For  $P_1$  to  $P_4$ , calculate the output  $Y^{output}$ .

Step3. Update weights by  $w_i = w_i + \Delta w_i$   

$$\Delta w_i = 2 * \alpha (Y - Y^{output}) \frac{\partial Y^{output}}{\partial w_i}$$
  
 $\alpha$  is a small constant

Step4. Repeat the above step until no more to update.

### Simplest neural network



Input nodes

Output nodes

### From Logistic regression to Neural Network

Advantages	Fast prediction.
	Successful in real-life problems
	High tolerance to noisy data
Disadvantages	Long training time
	Poor interpretability

✧ Alphafold: the most successful deep learning application.

## 2. Binary classification evaluation

Which clustering method is better? Which classification should we trust?

➔ Quantitative values are needed to summarise the performance of different methods.

✧ Confusion matrix

		Predicted class	
		Class=Yes	Class=No
Actual class	Class=Yes	a(TP)	b(FN)
	Class=No	c(FP)	d(TN)

Most widely-used metric:

$$\text{Accuracy} = \frac{a + d}{a + b + c + d} = \frac{TP + TN}{TP + TN + FP + FN}$$

-However, there may be misleading for imbalanced data.

		Predicted class	
		Class=Yes	Class=No
Actual class	Class=Yes	4949(TP)	0(FN)
	Class=No	51(FP)	0(TN)

**Imbalanced classes**

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} = \frac{4949}{4949 + 51} = 0.99$$

Maybe misleading for **imbalanced data**

-Although we have **precision** (how many are correct among the predicted positive samples), **recall** (how many actual positive samples are predicted to be positive) and **F1 score** (the weight average of precision and recall) -> Still maybe misleading for imbalanced data.

		Predicted class	
		Class=Yes	Class=No
Actual class	Class=Yes	4949(TP)	0(FN)
	Class=No	51(FP)	0(TN)

$$Precision = \frac{a}{a + c} = \frac{4949}{4949 + 51} = 0.99$$

$$Recall = \frac{a}{a + b} = 1$$

$$F1\ score = \frac{2 * precision * recall}{precision + recall} = 0.995$$

		Predicted class	
		Class=Yes	Class=No
Actual class	Class=Yes	4949(TP)	0(FN)
	Class=No	51(FP)	0(TN)

\*Look at the confusion matrix directly if you know it is an imbalanced dataset

$$Balanced\ accuracy = 0.5 * \left( \frac{TP}{TP + FN} + \frac{TN}{TN + FP} \right) = 0.5$$

➤ Value is not absolute. Context matters.

### 3. Cross-validation

✧ A technique for assessing how the result of a machine learning analysis will generalise to an independent data set -> measure the performance of models.

For one round cross-validation: Partitioning a set of data into complementary subsets.

Performing the analysis on one subset (training set).

Validating the analysis on the other subset (testing set).

**n-fold cross-validation** (train multiple times, leave out a disjoint subset of data each time for validation. Average the validation set accuracies)

### Process:

- Randomly partition data into n disjoint subsets
- For  $i = 1$  to  $n$ 
  - Validation Data =  $i$ -th subset
  - $h \leftarrow$  classifier trained on all data **except for Validation Data**
  - Accuracy( $i$ ) = accuracy of  $h$  on Validation Data
- Final Accuracy = **mean** of the  $n$  recorded accuracies

Examples: 5-fold cross-validation

10 data points:

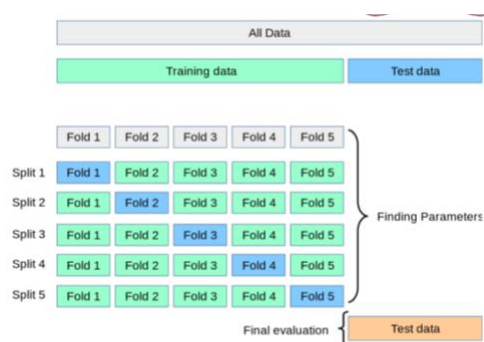
- P1-P10

5-fold

- P1-2, P3-4, P5-6, P7-8, P9-10
- The grouping can be **random**

### Procedure

- P1-2's results based on the model from P3-10
- ...
- P9-10's results based on the model from P1-8
- Averaging



**Leave-one-out cross-validation** (a special case of  $n$ -fold cross-validation, where  $n=N$ )

### Process:

- Partition data into  $N$  disjoint subsets, each containing one data point
- For  $i = 1$  to  $N$ 
  - Validation Data =  $i$ -th subset
  - $h \leftarrow$  classifier trained on all data **except for Validation Data**
  - Accuracy( $i$ ) = accuracy of  $h$  on Validation Data
- Final Accuracy = **mean** of the  $N$  recorded accuracies

## 4. Multi-class classification

✧ No need to change the algorithm for KNN but for logistic regression, we need to

1. Build a logistic regression for each class
2. When predicting, assign class with the highest value

3. When training, train  $3 \cdot N$  parameters, where  $N$ =number of classes we have.

4. Considering each class as a binary classification problem.

Example:

$$\text{Macro - average} = \frac{0.9 + 0.95 + \dots + 0.7 + 0.2}{6} = 0.73$$

$$\text{Micro - average} = \frac{0.9 * 150 + \dots + 0.2 * 10}{150 + \dots + 10} = 0.85$$

Class	Accuracy	Cells
1	0.9	150
2	0.95	50
3	0.85	100
4	0.8	40
5	0.7	20
6	0.2	10

The low-performance of small classes will show up in Macro-average

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